Answer Sheet to the Written Exam

Corporate Finance and Incentives

December 2020

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student is expected to be able to:

Knowledge:

1. Identify, describe and discuss financial problems encountered by firms and investors,
2. Account for and understand the core models and methodologies in the field of Financial Economics,
3. Define the core concepts of Financial Economics,
4. Criticize and reflect upon the main models in Finance, relating them to current issues in financial markets and corporate finance.

Skills:

1. Select and apply core models and methodologies to analyse standard problems in Finance, partly using Excel,
2. Master the analysis of given problems, assessing models and results, putting results into perspective,
3. Argue about financial problems and issues in a scientific and professional manner, drawing upon the relevant knowledge of the field.

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems,
2. Select and evaluate solutions to complex, unpredictable situations in financial markets or corporations,
3. Approach more advanced models, methodologies and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1–3, skills 1–3, competencies 1 and 2. Problem 4 emphasizes knowledge points 1–4, skill 3, and competencies 1 and 3.

Some numerical calculations may differ slightly depending on the commands chosen for computation, so a little slack is allowed when grading the answers.

Problem 1 (CAPM 30%)

We provide you with 60 monthly return observations for three assets (*AAPL*, *MMM* and *BA*) in a separate file named *data_CFIDec2020.xls*. The returns are stated in percent. Use the data and Excel to complete the following tasks:

1) *Compute the sample variance-covariance matrix and sample mean of the returns for the three assets.* Computation yields

$$\mu = E(r) = \begin{pmatrix} 3.109 \\ 0.620 \\ 1.621 \end{pmatrix} \tag{1}$$

where r is the vector of returns for *AAPL, MMM,* and *BA*.

The sample covariance matrix looks like

$$\Sigma = \text{Cov}(r) \begin{pmatrix} 74.814 & 18.912 & 26.700 \\ 18.912 & 33.981 & 27.785 \\ 26.700 & 27.785 & 150.543 \end{pmatrix}. \tag{2}$$

2) *Find the minimum variance portfolio weights of the risky assets and calculate the expectation and volatility of this portfolio's return.* The variance of a portfolio with weights w is

$$\sigma^2_{\text{pf}} = w'\Sigma w.$$

The minimum variance portfolio weights are the minimizing argument of

$$w_{\text{mvp}} = \arg\min w'\Sigma w \text{ s.t. } w'\iota = 1$$

where ι is a vector of ones. w_{mvp} can be derived analytically as

$$w_{\text{mvp}} = \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota$$

In our case, we get

$$\Sigma^{-1} = \begin{pmatrix} 0.016 & -0.008 & -0.001 \\ -0.008 & 0.038 & -0.006 \\ -0.001 & -0.006 & 0.008 \end{pmatrix}$$

and therefore

$$w_{\text{mvp}} = \begin{pmatrix} 0.207 \\ 0.767 \\ 0.026 \end{pmatrix}.$$

The expected return of a portfolio is $E(r_{\text{pf}}) = w'\mu$ and the volatility is the square root of the variance. In the case from above this is just:

$$E(r_{\text{mvp}}) = 1.161$$

and

$$\sigma(r_{\text{mvp}}) = \sqrt{w'_{\text{mvp}}\Sigma w_{\text{mvp}}} = 5.540$$

3) *Compute the efficient tangent portfolio weights for a risk-free rate* $r_f = 0.1\%$. We derived the tangency portfolio weights w^* in Slide 21 of the third lecture as

$$w^* = \frac{1}{\iota'\Sigma^{-1}(\mu - r_f)}\Sigma^{-1}(\mu - r_f) = \begin{pmatrix} 1.199 \\ -0.341 \\ 0.143 \end{pmatrix}$$

4) *Explain the Two Mutual Fund Theorem.* The Two Mutual Fund Theorem states that any combination (a portfolio) of two efficient portfolios is again efficient. As a result, only two distinct portfolios are required to characterize the entire efficient frontier.

5) *Compute the beta of asset 3 (BA) with respect to the efficient tangent portfolio. Which portfolio weights deliver the lowest possible volatility for an investment that has the same expected return as investing in asset 3 (BA) only (you can assume that investing and borrowing at the risk-free rate* $r_f = 0.1\%$ *is possible)?*

The beta can be computed as the regression coefficient of a regression of $r_{\text{BA}} - r_f$ on the portfolio returns of the efficient portfolio $w^{*\prime}r$ which yields 0.417. Alternatively, you could make use of the CAPM equation and compute

$$\beta = E(r_{\text{BA}} - r_f)/E(r_{\text{eff}} - r_f) = 0.406.$$

Next you can make use of the two-mutual fund theorem: You know that an investment in the risk-free rate is efficient, as is an investment in the efficient tangent portfolio. Also, we know that a portfolio that invests β in the efficient tangent portfolio and $1 - \beta$ in the risk free rate just earns

$$E(r_p) = \beta E(w^{*\prime}\mu) + (1 - \beta)r_f = r_f + \beta E(w^{*\prime}\mu - r_f)$$

which is the expected return of BA. Therefore, a portfolio that invests $1 - 0.417$ in the risk-free rate and βw^* in the efficient tangent portfolio is efficient and thus delivers the lowest possible volatility for an investment with the same expected return as BA. The resulting volatility is 4.284.

Problem 2 (Corporate Finance 20%)

1) The assets provide $70\%20 = 14$ after corporate tax. This cash-flow stream is a perpetuity valued at $14/(2\%) = 700$.

2) The perpetual debt provides only interest in all future. This interest can be deducted from the tax base, so $30\%50 = 15$ is the present value of the tax shield.

3) The value of the firm is now $700 + 15 = 715$. Note that the 50 raised in debt is passed on to shareholders.

4) In order to raise 50 from creditors, the firm must promise future cash flow which is valued such by creditors are paying their personal tax. When 50 is sent to shareholders through a repurchase they may be taxed more leniently than if this value were to be distributed as future dividends (Berk and DeMarzo, section 17.3). To determine the overall gain to investors, the firm could study the total value of taxes paid by the firm, creditors and shareholders.

Problem 3 (Options 25%)

1) *Compute the risk-neutral probabilities for each branch of the tree.* Risk-neutral pricing implies a martingale property for the price of the asset. Therefore, at each node i we can solve for the risk-neutral probability π_i of high returns by solving for

$$S_i = \frac{1}{1+r_f}\left(\pi_i S_i^U + (1-\pi_i)S_i^D\right) = \frac{1}{1+r_f} S_i\left(\pi_i(r_U - r_D) + r_D\right)$$

where $r_U = 1.2$ and $r_D = 0.8$ in the exercise. It becomes immediately clear that the risk-neutral probabilities are the same at each node, such that we can denote

$$\pi = \frac{(1 + r_f - r_d)}{r_U - r_D} = 0.525.$$

2) *Consider two European options expiring at time 2: one Call option with strike price 96 and one Put option with strike price 96. Compute the market values (i.e. premiums) of the Call, C_0, and the Put, P_0 at time 0.* We use the risk-neutral probability from a) to derive the market value of the two options. Note first, that the Call option with strike price 96 pays out 0 in states S_2 and S_2^D and $144 - 96 = 48$ in state S_2^U. Thus, the arbitrage-free market price of the Call is

$$C_0 = \frac{1}{(1+r_f)^2}\pi^2 48 = 12.969.$$

Similar for the Put which pays out $96 - 64 = 32$ in state S_2^D and 0 in both other states. The market value is then

$$P_0 = \frac{1}{(1+r_f)^2}(1-\pi)^2 32 = 7.078.$$

3) *What is the arbitrage-free value of a straddle (simultaneously buying both a put option and a call option for the underlying security with the same strike price 96 and the same expiration date, time 2)? What are the cash flows at time 2 in each state if you finance the portfolio by borrowing at the risk-free rate?* The market price is simply the sum of $C_0 + P_0 = 20.047$. If you finance the straddle by borrowing 20.047 at time 0, the resulting

(negative) cash flow at time 2 is $20.047(1+r_f) = 20.449$. As a result, the portfolio pays out $48 - 20.449 = 27.56$ in state S_2^U, -20.449 in state S_2 and $32 - 20.449 = 11.56$ in state S_2^D.

4) *Argue why the (risk-neutral) expected payoff of the portfolio in question 3) (buying the straddle and financing at the risk-free rate) at time* 2 *has to be zero.* First verify this result given the payouts in each state:

$$\pi^2 27.56 + (1-\pi)^2 11.56 - 2\pi(1-\pi))20.449 \approx 0$$

This holds simply because the market is arbitrage-free. If the risk-neutral expected return would not be zero then a (self-financing) portfolio would generate positive expected returns which corresponds to an arbitrage opportunity. Such an existence was ruled out by the fundamental theorem of asset pricing.

Problem 4 (Various Themes 25%)

1) Section 13.2 in Berk and DeMarzo lays out some consequences.

2) A good answer could discuss points such as the following. When uncertainty is high, as measured by this new index, it is plausible that risk-averse investors demand a high risk premium. With reference to chapter 12 in Berk and DeMarzo, then the cost of capital for firms is high, and firms can be expected to find fewer projects of positive net present value. Then they invest less. With reference to chapter 22, when uncertainty is high, it may seem attractive for firms to postpone investments in order to learn more about their profitability before committing capital.

3) The claim appears on page 1068 in Berk and DeMarzo. See the explanation and discussion in section 30.2.